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Fixed points of non-self almost contractions

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ABSTRACT.

Let X be a convex metric space, K a non-empty closed subset of X and $T : K \to X$ a non-self almost contraction. Berinde and Păcurar [Berinde, V. and Păcurar, M., *Fixed point theorems for nonself single-valued almost contractions*, Fixed Point Theory, **14** (2013), No. 2, 301–312], proved that if T has the so called property (M) and satisfies Rothe's boundary condition, i.e., maps ∂K (the boundary of K) into K, then T has a fixed point in K. In this paper we observe that property (M) can be removed and, hence, the above fixed point theorem takes place in a different setting.

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