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On an isomorphism lying behind the class number formula

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ABSTRACT.

Let p be an odd prime such that the Greenberg conjecture holds for the maximal real cyclotomic subfield $\mathbb{K}_1$ of $\mathbb{Q}[\zeta_p]$. Let $A_n = (\mathcal{C}(\mathbb{K}_n))_p$ be the p-part of the class group of $\mathbb{K}_n$, the n-th field in the cyclotomic tower, and let $\underline{E}_n$, $\underline{C}_n$ be the global and cyclotomic units of $\mathbb{K}_n$, respectively. We prove that under this premise, there is some n_0 such that for all $m \geq n_0$, the class number formula $\left|(\underline{E}_m/\underline{C}_m)_p\right| = |A_m|$ hides in fact an isomorphism of $\Lambda[\mathrm{Gal}(\mathbb{K}_1/\mathbb{Q})]$-modules.

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