Dedicated to Prof. Hong-Kun Xu on the occasion of his 60th anniversary

A generalization of the (CN) inequality and its applications

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ABSTRACT.

We extend the (CN) inequality of Bruhat and Tits in CAT(0) spaces to the general setting of uniformly convex hyperbolic spaces. We also show that, under some appropriate conditions, the sequence of Ishikawa iteration defined by Panyanak converges to a strict fixed point of a multi-valued Suzuki mapping.

REFERENCES

[1] Abkar, A. and Eslamian, M., *Fixed point theorems for Suzuki generalized nonexpansive multivalued mappings in Banach spaces*, Fixed Point Thoery Appl. Art., **2010**, Art. ID 457935 (2010), 1–10

[2] Bauschke, H. H. and Combettes, P. L., *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*, Springer, New York, 2011

[3] Bridson, M. and Haefliger, A., *Metric Spaces of Non-Positive Curvature*, Springer, Berlin, 1999

[4] Bruhat, F. and and Tits, J., *Groupes réductifs sur un corps local. I. Données radicielles valuées*, Inst. Hautes Études Sci. Publ. Math., **41** (1972), 5–251

[5] Chuadchawna, P., Farajzadeh, A. and Kaewcharoen, A., *Convergence theorems and approximating endpoints for multivalued Suzuki mappings in hyperbolic spaces*, J. Comp. Anal. Appl., **28** (2020), 903–916

[6] Dhompongsa, S. and Panyanak, B., *On $\Delta-$convergence theorems in $CAT(0)$ spaces*, Comput. Math. Appl., **56** (2008), 2572–2579

[7] Espinola, R., Lorenzo, P. and Nicolae, A., *Fixed points, selections and common fixed points for nonexpansive-type mappings*, J. Math. Anal. Appl., **382** (2011), 503–515

[8] Garcia-Falset, J., Lorens-Fuster, E. and Moreno-Gálvez, E., *Fixed point theory for multivalued generalized nonexpansive mappings*, Appl. Anal. Discrete Math., **6** (2012), 265–286

[9] Garcia-Falset, J., Lorens-Fuster, E. and Suzuki, T., *Fixed point theory for a class of generalized nonexpansive mappings*, J. Math. Anal. Appl., **375** (2011), 185–195

[10] Ibn Dehaish, B. A., Khamsi, M. A. and Khan, A. R., *Mann iteration process for asymptotic pointwise nonexpansive mappings in metric spaces*, J. Math. Anal. Appl., **397** (2013), 861–868

[11] Ishikawa, S., *Fixed points by a new iteration method*, Proc. Amer. Math. Soc., **44** (1974), 147–150

[12] Khamsi, M. A. and Khan, A. R., *Inequalities in metric spaces with applications*, Nonlinear Anal., **74** (2011), 4036–4045

[13] Kirk, W. A. and Panyanak, B., *A concept of convergence in geodesic spaces*, Nonlinear Anal., **68** (2008), 3689–3696

[14] Kudtha, A. and Panyanak, B., *Common endpoints for Suzuki mappings in uniformly convex hyperbolic spaces*, Thai J. Math., **Special issue** (2018), 159–168

[15] Leustean, L., *A quadratic rate of asymptotic regularity for $CAT(0)$-spaces*, J. Math. Anal. Appl., **325** (2007), 386–399

[16] Leustean, L., *Nonexpansive iterations in uniformly convex W-hyperbolic spaces*, in Nonlinear Analysis and Optimization I. Nonlinear Analysis, vol. 513 of Contemporary Mathematics, pp. 193-210, American Mathematical Society, Providence, RI, USA, 2010

[17] Laowang, W. and Panyanak, B., *Common fixed points for some generalized multivalued nonexpansive mappings in uniformly convex metric spaces*, Fixed Point Theory Appl., **2011**:20 (2011), 1–9

[18] Mann, W. R., *Mean value methods in iteration*, Proc. Amer. Math. Soc., **4** (1953), 506–510

[19] Panyanak, B., *Approximating endpoints of multi-valued nonexpansive mappings in Banach spaces*, J. Fixed Point Theory Appl., **20** (2018), No. 2, Art. 77, 8 pp.

[20] Panyanak, B., *Mann and Ishikawa iterative processes for multivalued mappings in Banach spaces*, Comput. Math. Appl., **54** (2007), 872–877

[21] Panyanak, B., *On an open problem of Kyung Soo Kim*, Fixed Point Theory Appl., 2015, 2015:186, 12 pp.

[22] Xu, H. K., *Inequalities in Banach spaces with applications*, Nonlinear Anal., **16** (1991), 1127–1138

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