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Dedicated to Prof. Hong-Kun Xu on the occasion of his 60^{th} *anniversary*

An inertial extragradient method for solving bilevel equilibrium problems

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ABSTRACT.

In this paper, we propose an algorithm with two inertial term extrapolation steps for solving bilevel equilibrium problem in a real Hilbert space. The inertial term extrapolation step is introduced to speed up the rate of convergence of the iteration process. Under some sufficient assumptions on the bifunctions involving pseudomonotone and Lipschitz-type conditions, we obtain the strong convergence of the iterative sequence generated by the proposed algorithm. A numerical experiment is performed to illustrate the numerical behavior of the algorithm and also comparison with some other related algorithms in the literature.

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