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Stability of oscillatory solutions of impulsive differential equations with piecewise alternately advanced and retarded argument of generalized type

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ABSTRACT. In this study, we investigate scalar impulsive advanced and delayed differential equations with piecewise constant argument of generalized type, abbreviated as IDEPCAG, where the arguments are represented as general step functions. We propose criteria for the existence of oscillatory and non-oscillatory solutions, and derive sufficient conditions for the stability of the zero solution. Our results are novel, and extend and improve upon previous publications. Additionally, we provide several numerical examples and simulations to demonstrate the feasibility of our findings.

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