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Stability and bifurcations in an epidemic model with nonlinear transmission and removal rates

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ABSTRACT. A generalized SEIR epidemiological model, incorporating general nonlinear transmission and removal rates, has been developed and investigated. Local and global stability theory and bifurcation theory are used to determine the dynamics of the model. The presence of unique or coexisting attractors is proved and different scenarios for the evolution of the model, towards a stable equilibrium point or a limit cycle are found. The theoretical results are supported by numerical simulations, obtained using Holling type II functions.

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