

# On statistical convergence of topological Henstock-Kurzweil integral

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**ABSTRACT.** In this paper, we introduce Henstock-Kurzweil type integrable function (in brief, topological Henstock-Kurzweil integrable function) on a topological vector space associate with a Radon measure  $\mu$ . Basic results of topological Henstock-Kurzweil integrable function are discussed. Also, the relationship between topological Henstock-Kurzweil integral and Lebesgue integral is discussed. Moreover, we investigate several convergence theorems for  $\mu$ -measurable topological Henstock-Kurzweil integrable function on a topological vector space. Finally, we extent the notion of statistical convergence for topological Henstock-Kurzweil integrable function on a  $\mu$ -subcell of a topological vector space.

## ACKNOWLEDGMENTS

The authors thank to the Journal editors and reviewers for their constructive suggestions for better presentation of the article.

## REFERENCES

- [1] Corrao, G. *An Henstock-Kurzweil type integral on a measure metric space*. Doctoral Thesis, Universita Degli Studi Di Palermo, Palermo, 2013.
- [2] Duman, O.; Orhan, C.  $\mu$ -statistically convergent function sequences. *Czec. Math. J.* **54** (2004), no. 2, 413–422.
- [3] Fast, H. Sur la convergence statistique. *Colloq. Math.* **2** (1951), no. 3, 241–244.
- [4] Fridy, J. A. On statistical convergence. *Analysis*. **5** (1985), no. 4, 301–313.
- [5] Fridy, J. A. Statistical limit points. *Proc. Amer. Math. Soc.* **118** (1993), no. 4, 1187–1192.
- [6] Fridy, J. A.; Miller, H. I. A matrix characterization of statistical convergence. *Analysis*. **11** (1991), no. 1, 59–66.
- [7] Fridy, J. A.; Orhan, C. Lacunary statistical convergence. *Pacific J. Math.* **160** (1993), no. 1, 43–51.
- [8] Gardon, R. G. *The Integrals of Lebesgue, Denjoy, Perron, and Henstock*. American Mathematical Society, 1994.
- [9] Giuseppe, D. M.; Kocinac, L. D. R. Statistical convergence in topology. *Topol Appl.* **156** (2008), no. 1, 28–45.
- [10] Henstock, R. *The general theory of integration*. Oxford University Press, Oxford, UK, 1991.
- [11] Kalita, H.; Hazarika, B. A convergence theorem for ap-Henstock-Kurzweil integral and its relation to topology. *Filomat*. **36** (2022), no. 20, 1–10.
- [12] Kalita, H.; Hazarika, B.; Becerra, T. P. On AP-Henstock-Kurzweil Integrals and Non-Atomic Radon Measure. *Mathematics*. **11** (2023), no. 6, 1–16.
- [13] Kalita, H.; Agarwal, R. P.; Hazarika, B. Convergence of ap-Henstock-Kurzweil integral on locally compact spaces. *Czec. Math. J.* (2023), <https://doi.org/10.21136/CMJ.2023.0450-22>.
- [14] Klein, Ch.; Rolewicz, S. On Riemann integration of functions with values in topological linear spaces. *Studia Mathematica*. **80** (1984), no. 4, 109–118.
- [15] Paluga, R.; Canoy Jr, S. The Henstock Integral in topological vector spaces. *Mat. Matematika*. **24** (2001), no. 3, 34–47.
- [16] Sokolowski, J.; Zochowski, A. On the Topological Derivatives in Shape Optmization. *SIAM. J. Control and Optimization*. **37** (1999), no. 4, 1–35.
- [17] Srivastava, H. M.; Jena, B. B.; Paikray, S. K. Statistical Riemann and Lebesgue Integrable Sequence of Functions with Korovkin Type Approximation Theorems. *Axioms*. **10** (2021), no. 10, 1–16.

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Received: 22.01.2024. In revised form: 20.07.2024. Accepted: 14.09.2024  
2020 *Mathematics Subject Classification*. 26A39, 26A46, 54A20, 54B20.

Key words and phrases. Topological Henstock-Kurzweil integral, Cauchy's criterion, statistical convergence, topological vector spaces.

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- [18] Yeong, L. T. *Henstock-Kurzweil integration on Euclidean spaces*. Series in Real Analysis, no. 12, World Scientific, 2011.

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