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Cut-through connections of graphs

DANIELA MATISOVÁ[1] AND JURAJ VALISKA[2]

ABSTRACT. A 4-regular plane graph G is cut-through connected if any two vertices of G are connected by a cut-through path (that is, the path with the property that every two consecutive edges are not consecutive in local rotation of their common vertex). In this paper, we present the complete characterization of cut-through connected 4-regular plane graphs in terms of Gauss Codes or Extended Gauss Codes.

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Corresponding author: JURAJ VALISKA; juraj.valiska@tuke.sk

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[1]P. J. ŠAFÁRIK UNIVERSITY
INSTITUTE OF MATHEMATICS
JESENNÁ 5, 040 01 KOŠICE, SLOVAKIA
Email address: `daniela.matisova@student.upjs.sk`

[2]TECHNICAL UNIVERSITY IN KOŠICE
INSTITUTE OF MATHEMATICS AND THEORETICAL INFORMATICS
LETNÁ 9, 042 00 KOŠICE, SLOVAKIA
Email address: `juraj.valiska@tuke.sk`