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Unified Convergence Analysis of Certain At Least Fifth Order Methods

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ABSTRACT. A class of iterative methods was developed by Xiao and Yin in 2015 and obtained convergence order five using Taylor expansion. They had imposed the conditions on the derivatives of the involved operator of order at least up to four. In this paper, the order of convergence is achieved by imposing conditions only on the first two derivatives of the operator involved. The assumptions under consideration are weaker and the analysis is done in the more general setting of Banach spaces without using Taylor series expansion. The semi-local convergence analysis is also given. Further, the theory is justified by numerical examples.

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REFERENCES

[1] Argyros, I.K. *The theory and applications of iteration methods*. 2nd ed., Engineering Series CRC Press, Taylor and Francis Group: Boca Raton, FL, USA, 2022.
[2] Argyros, I.K.; Magréñan, A.A. *A contemporary study of iterative schemes*. Elsevier (Academic Press), New York, 2018.
[3] Berinde, V. *Iterative Approximation of Fixed Points*. Springer Berlin, Heidelberg, 2007.
[4] Blanchard, P. Complex dynamics on Riemann sphere. *Bull. Amer. Math. Soc.* **11** (1984), 85-141.
[5] Catinas, E. A survey on the high convergence orders and computational convergence orders of sequences. *Appl. Math. Comput.* **343** (2019), 1-20.
[6] Catinas, E. How many steps still left to x*? *SIAM review.* **63** (2021), no. 3, 585-624.
[7] Cordero, A.; Torregrosa, J.R. Variants of Newton's method using fifth order quadrature formulas. *Appl. Math. Comput.* **190** (2007), 686-698.
[8] Cordero, A.; Hueso, J.L.; Martínez, E.; Torregrosa, J.R. Increasing the convergence order of an iterative method for nonlinear systems. *Appl. Math. Lett.* **25** (2012), 2369-2374.
[9] Cordero, A.; Martínez, E.; Toregrossa, J.R. Iterative methods of order four and five for systems of nonlinear equations. *J. Comput. Appl. Math.* **231** (2012), 541-551.
[10] Darvishi, M.T.; Barati, A. A fourth-order method from quadrature formulae to solve systems of nonlinear equations. *Appl. Math. Comput.* **188** (2007), 257-261.
[11] Darvishi, M.T.; Barati, A. A third-order Newton-type method to solve systems of nonlinear equations. *Appl. Math. Comput.* **187** (2007), 630-635.
[12] Frontini, M.; Sormani, E. Third-order methods from quadrature formulae for solving systems of nonlinear equations. *Appl. Math. Comput.* **149** (2004), 771-782.
[13] Liu, Z.; Zheng, Q.; Huang, C. Third- and fifth-order Newton-Gauss methods for solving nonlinear equations with n variables. *Appl. Math. Comput.* **290** (2016), 250-257.
[14] Magréñan, A.A.; Gutiérrez, J.M. Real dynamics for damped Newton's method applied to cubic polynomials. *J. Comput. Appl. Math.* **275** (2015), 527-538.

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[15] Magréñan, A.A.; Argyros, I.K.; Rainer, J.J.; Sicilia, J.A. Ball convergence of a sixth-order Newton-like method based on means under weak conditions. *J. Math. Chem.* **56** (2018), 2117-2131. https://doi.org/10.1007/s10910-018-0856-y.

[16] Ortega, J.M.; Rheinboldt, W.C. *Iterative solution of nonlinear equations in several variables*. Volume 30 of Classics in Applied Mathematics Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA. Reprint of the 1970 original, 2000.

[17] Sadananda, R.; George, S.; Kunnarath, A.; Padikkal, J.; Argyros, I.K. Enhancing the practicality of Newton-Cotes iterative method. *J Appl Math Comput.* **69** (2023), 3359-3389.

[18] Sharma, J.R.; Gupta, P. An efficient fifth order method for solving systems of nonlinear equations. *Comput. Math. Appl.* **67** (2014), 591-601.

[19] Traub, J.F. Iterative methods for the solution of equations. Prentice-Hall Series in Automatic Computation, Prentice-Hall, Englewood Cliffs, N.J, 1964.

[20] Varona, J.L. Graphic and numerical comparison between iterative methods. *The Math. Intelligencer* **24** (2002), no. 3, 37-46.

[21] Weerakoon, S.; Fernando, T. A variant of Newton's method with accelerated third-order convergence. *Appl. Math. Lett.* **13** (2000), no. 8, 87-93.

[22] Xiao, X.Y.; Yin, H.W. A new class of methods with higher order of convergence for solving systems of nonlinear equations. *Appl. Math. Comput.* **264** (2015), 300-309. https://doi.org/10.1016/j.amc.2015.04.094.

[23] Xiao, X.Y.; Yin, H.W. Increasing the order of convergence for iterative methods to solve nonlinear systems. *Calcolo* **53** (2016), 285-300.

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