

Global-fixed-point property of gyrogroup actions

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ABSTRACT. The notion of a gyrogroup action generalizes that of a group action. This work, inspired by the Extended Cartan Fixed Point Theorem, is devoted to a fixed-point property of gyrogroup actions. In the case when a gyrogroup G acts on a non-empty set X , X is said to have the global-fixed-point property if there exists an element x in X such that $a \cdot x = x$ for all $a \in G$. In this paper, several conditions for X to have the global-fixed-point property are determined. A few examples regarding the results are also discussed.

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