

Two generalized cyclic projection algorithms for solving a class of the split feasibility problem in real Hilbert spaces

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ABSTRACT. In the present paper, we propose two new cyclic projection algorithms for solving a class of the split feasibility problem and analyse their strong convergence. Our algorithms are based on the hybrid or shrinking projections methods and use the general index control mapping. Our algorithm can be implemented without any need for information about the norm of the transfer mappings or the cost operators' inverse strong monotone coefficient.

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