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Second-order necessary conditions for bilevel programs via KKT reformulation

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ABSTRACT. The KKT reformulation is widely recognized as a prominent approach for analyzing bilevel optimization problems, especially in the case of convex lower-level problems. This paper focuses on exploring the KKT reformulation and deriving second-order necessary optimality conditions for local solutions to bilevel programs. The derived optimality conditions are formulated within the framework of recently established constraint qualifications for nonlinear optimization problems. Specifically, three types of second-order necessary optimality conditions are presented based on Clarke, Mordukhovich and strong stationarity conditions.

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