

On a wave equation with mixed dynamic boundary conditions

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ABSTRACT. Let $d \in \mathbb{N}$, $T > 0$ and $\Omega \subset \mathbb{R}^d$ be open bounded with Lipschitz boundary $\partial\Omega = \Gamma_0 \cup \Gamma_1$. Consider the wave equation

$$(P) \quad \begin{cases} u_{tt} - \Delta u_{tt} - \Delta u = k(t) |u|^{p-1} u & \text{in } \Omega \times (0, T), \\ u = 0 & \text{on } \Gamma_0 \times (0, T), \\ \frac{\partial u_{tt}}{\partial \nu} + \frac{\partial u}{\partial \nu} = \omega |u|^{m-1} u & \text{on } \Gamma_1 \times (0, T), \\ u(0) = u_0, \quad u_t(0) = u_1 & \text{in } \Omega. \end{cases}$$

Under suitable conditions on the indices p , m and the initial functions u_0 , u_1 , we investigate the existence and uniqueness of a global weak solution to (P) together with the decaying and blow-up properties using the potential well method.

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