

# Third-Order Neutral Difference Equations: Improved Oscillation Criteria via Linearization Method

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ABSTRACT. This paper studies the oscillatory and asymptotic behavior of solutions to the third-order neutral delay difference equation

$$\Delta (\mu(\nu)[\Delta^2\alpha(\nu)]^a) + b(\nu)\beta^a(\nu - \tau) = 0,$$

where  $\alpha(\nu) = \beta(\nu) + \varrho\beta(\nu - \sigma)$ , using linearization method and then comparing with the second-order delay difference equations whose oscillatory properties are known. The obtained criteria are new, improve and extend some of the known results. This is verified by means of two specific examples.

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