

Existence and uniqueness of solution for a class of superlinear Kirchhoff-type equations on the real half-line

ISHAK KETTAF AND SOFIANE KHOUTIR

ABSTRACT. This paper is concerned with a class of Kirchhoff-type equations on the real half-line. By using the Mountain Pass theorem, we show the existence and uniqueness of solution in the case where f is a superlinear function. Our results improve and extend a recently published ones.

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Corresponding author: Sofiane Khoutir; skhoutir@usthb.dz

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FACULTY OF MATHEMATICS

AMNEDP LABORATORY

USTHB, PB 32 EL-ALIA, BAB EZZOUAR 16111, ALGIERS, ALGERIA

Email address: ikettaf@usthb.dz

Email address: skhoutir@usthb.dz